Vaporization of Liquid-Air Droplets by High-Power Laser Beams

Kuei-Yuan Chien*
Naval Surface Weapons Center, Silver Spring, Md.

The interaction between a high-power laser beam and a moving cloud of vaporizing droplets was considered theoretically. The study was performed to assess partially the feasibility of a new concept of generating an ultra-high-total-enthalpy flow, as was recently proposed by Johnson and von Ohain. The spectral absorption coefficient of liquid air was determined at the fundamental vibration-rotation frequencies of liquid nitrogen and oxygen. A perturbation solution was sought for a small but arbitrary variation in the velocity of the moving droplets. The first-order correction to the uniform-velocity solution of Glickler has been explicitly carried out in closed form. Results indicate that the concept of vaporizing liquid-air droplets by laser beams is feasible, although the power required, according to our present knowledge of the radiative properties of liquid air, is high. The idea of mixing strongly absorbing to reduce the total power required appears quite attractive.

Nomenclature

d = radiative decay length
E_C = energy exchange ratio
f = dimensionless laser intensity
g = dimensionless droplet volume
h = dimensionless velocity perturbation
H = function defined by Eq. (22)
I = laser intensity
I_0 = incident laser intensity
L = heat of vaporization of air
P = function defined by Eq. (21)
Q_a = absorption cross section
$\bar{Q}$ = dimensionless quantity defined by Eq. (2)
R = droplet radius
R_0 = initial droplet radius
S = mean separation distance between droplets
u = flow velocity
V = droplet volume
X = distance measured in the flow direction
X_C = critical distance
y = distance measured normal to the flow direction
α = volumetric absorption coefficient of liquid air
ϵ = expansion parameter
θ = angle between incident and scattered beams
μ_g = viscosity of the carrier gas
ρ_ℓ = density of liquid air
τ_f = characteristic flow time
τ_O = characteristic evaporation time
τ_{RG} = characteristic evaporation time based on Rayleigh-Gans' formula, Eq. (8)

Introduction

A PROPER description of the interaction between a cloud of liquid droplets and a high-power laser beam is of great interest to problems such as fog clearing,[1-4] laser propagation in the atmosphere,[2,4] and vehicle protection against laser heating by particle injection. A new area of application is considered in the present paper, namely, the vaporization of liquid-air droplets by high-power laser beams. The study was performed to assess partially the feasibility of a new concept of generating an ultra-high-total-enthalpy flow, as was recently proposed by Johnson and von Ohain.[5] The concept employs a multicomponent-flow technique in which the kinetic energy is transferred from a low-molecular-weight carrier gas (such as H_2 or He) to a much-higher-molecular-weight gas (such as air). In the proposed direct-energy-transfer process, liquid-air droplets would be vaporized to yield the high-enthalpy airflow. Among the several possibilities suggested in Ref. 5, the idea of utilizing a high-power laser beam to achieve the desired vaporization appears very attractive. This is the subject of the present paper.

Absorption of laser radiation supplies the required energy to heat and vaporize the droplets. Therefore, we shall first discuss the spectral absorption coefficient of liquid air. The governing equations and the controlling parameters will then be described. A perturbation method will be used to systematically study the effect of a small but arbitrary variation in the velocity of the moving cloud of droplets. Finally, an example will be used to illustrate the level of the total power required, and several suggestions discussed for improving the efficiency of the energy transfer.

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*Research Aerospace Engineer, Applied Aerodynamics Divsion. Associate Fellow AIAA.

Analysis and Results

Absorption Coefficient of Liquid Air

Air is defined here as a mixture consisting of 79% N_2 and 21% O_2 by volume. In liquid or compressed gaseous form, both oxygen and nitrogen absorb radiation selectively at wavelengths of 6.4 and 4.3μ, respectively.[6,7] Furthermore, it was discovered that CO and NO lasers will lase near 6.4 μ, whereas HBr has been observed to lase near 4.3 μ.[8] Therefore, liquid-air droplets can be vaporized with two laser beams operating at these wavelengths.

Despite a rather extended period of study on the subject, no systematic measurements of the absorption coefficient of liquid air over wide enough temperature and pressure ranges are available in the literature. In view of this situation, the low-temperature data of Smith, Keller and Johnston[7] were employed to calculate the binary absorption coefficients of N_2 and O_2 at the corresponding fundamental vibration-

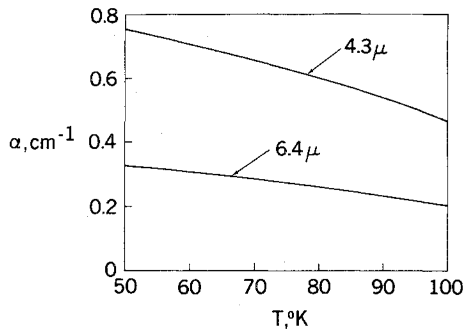

Fig. 1 Volumetric absorption coefficient of liquid air.

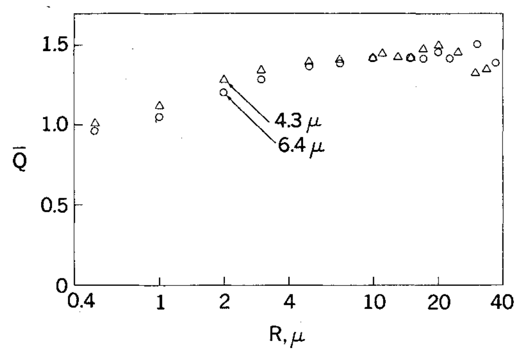

Fig. 2 Ratio of absorption cross section by Mie's theory to that by Rayleigh-Gans' theory.

rotation frequencies. Since air is a mixture, contributions from N_2 at 6.4 μ and from O_2 at 4.3 μ need to be included. The only quantitative spectral data of O_2-N_2 mixtures, reported by Shapiro and Gush[9] on the fundamental vibration-rotation band of O_2 around 6.4 μ at room temperature, suggested that O_2 and N_2 molecules were equally effective in inducing absorption of radiation. With no additional information available, the assumption was made that this is generally valid at low temperatures for both bands. We did not include any ternary term in the calculation of the absorption coefficient simply because there was no information on their magnitudes at the low-temperature range of interest to us. The volumetric absorption coefficient for liquid air evaluated at both wavelengths is shown in Fig. 1. For increasing temperature, the absorption coefficient decreases because the partial densities of N_2 and O_2 decrease.

Problem Formulation

Consider a two-phase flow comprising the carrier gas and the liquid-air droplets moving in the X-direction. A laser beam, directed perpendicular to the flow direction, is applied to the boundary of the two-phase flow, $y=0$. For the spectral regions under consideration, it is reasonable to assume that both the carrier gas, and the vaporized air are transparent to the laser radiation. Since the time required to raise a droplet from its initial temperature to the evaporation temperature as compared to the evaporation time is negligible,[4] droplets can be considered to be initially at the evaporation temperature. We shall assume that the incident laser beam is perfectly collimated and of uniform intensity. Furthermore, the disturbance caused by the vaporization of the droplets is assumed to be too weak to modify the prescribed flow velocity significantly. For simplicity, the following additional assumptions are made: 1) the problem is steady and two dimensional; 2) droplets are spherical, uniformly distributed, and (initially) identical in size; 3) both the heat conduction from the droplet surface and the kinetic energy of the vapor leaving the droplet are negligible as compared to the energy absorbed in the sphere. Assumption 2) may be partially relaxed by considering a distribution function for the droplet radius (see Ref. 4). Conditions under which assumption 3) is valid have been discussed by Williams.[1] For example, for an incident laser intensity of 10^6 w/cm^2 and a droplet temperature of 60K, calculations indicate that conduction is negligible if the droplet radius $R \gtrsim 3\mu$; whereas for $R \lesssim 50\mu$, the kinetic energy of the vapor is small.

Under these assumptions, the equation of energy conservation for a sphere is

$$u\,\partial V/\partial X = -IQ_a/\rho_\ell L \tag{1}$$

where u is the flow velocity in the X-direction, V the volume of the droplet, I the laser intensity, Q_a the absorption cross section, and L and ρ_ℓ are, respectively, heat of vaporization and density of liquid air. Q_a may be written as

$$Q_a = \bar{Q}\,\alpha\,V \tag{2}$$

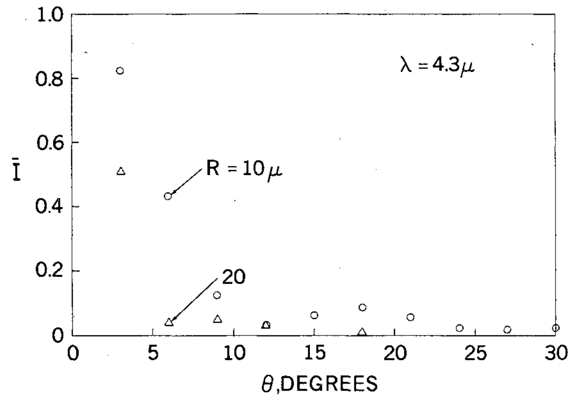

Fig. 3 Angular distribution of scattered intensity at 4.3μ.

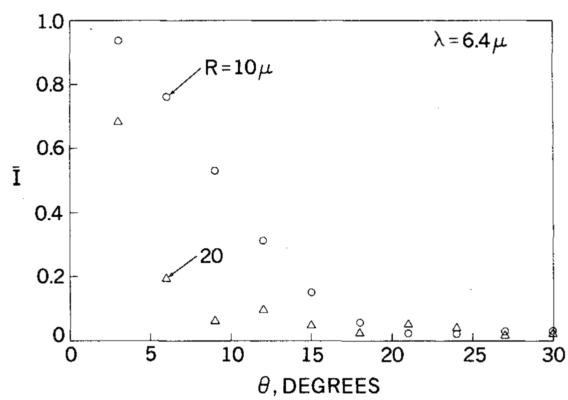

Fig. 4 Angular distribution of scattered intensity at 6.4μ.

where α is the volumetric absorption coefficient of the liquid air and $\bar{Q}$ the ratio of the exact Q_a to that given by the Rayleight-Gans' formula. Exact calculations based on Mie's theory[10] have been carried out by the numerical method of Howell.[11] Results for liquid-air droplets are shown in Fig. 2. Obviously, $\bar{Q}$ depends generally on the droplet size and, for $R > 1\mu$, is larger than unity. However, for $R > 5\mu$, $\bar{Q}$ is approximately size-independent. We shall take $\bar{Q}$ to be constant and equal to 1.4.

In general, the equation of radiative transfer contains terms of absorption, emission, and scattering. For the present problem, the emission term is negligible. The angular distribution of the scattered intensity normalized by that at $\theta=0$ as obtained from Mie's theory is shown in Fig. 3 and 4 (where θ is the angle between the incident and the scattered beams). Clearly, for $R \gtrsim 10\ \mu$, scattering is predominantly in the forward direction. Hence, as an approximation, we shall consider the limiting situation that scattering occurs only in the direction of $\theta=0$ (which is indistinguishable from the transmitted radiation). This approxiamtion simplifies the problem

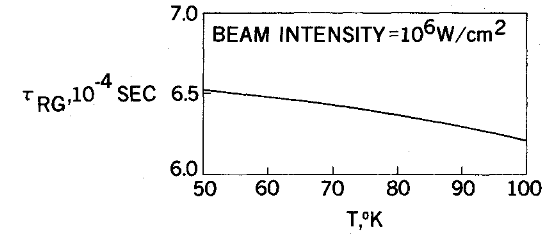

Fig. 5 Characteristic evaporation time of liquid air for $\bar{Q}=1$.

considerably. Thus, the equation of radiative transfer becomes

$$\partial I/\partial y = -\bar{Q}\,\alpha\,(2R/S)^3\,I \tag{3}$$

where S is the mean separation distance between droplets. The boundary conditions are

$$V(0,y) = (4\pi/3)R_0^3 \tag{4a}$$

and

$$I(X,0) = I_0 \tag{4b}$$

where R_0 is the initial droplet radius and I_0 the incident laser intensity. Consider a small but arbitrary perturbation in the flow velocity

$$u = u_0\,[1-\epsilon h\,(y)] \tag{5}$$

where u_0 and ϵ are constants, $h \sim 0(1)$ and $1 \gg \epsilon > 0$. We may define a characteristic time

$$\tau_o = 3\rho_\ell L/\bar{Q}\,\alpha\,I_0 \tag{6}$$

and a characteristic length

$$d = (1/\alpha\bar{Q})\,(S/2R_0)^3 \tag{7}$$

It is obvious that, without $\bar{Q}$, Eq. (6) is simply the characteristic evaporation time derived by Williams[1]

$$\tau_{RG} = 3\rho_\ell L/\alpha\,I_0 \tag{8}$$

which is based on the Rayleigh-Gans' formula for the absorption cross section. Equation (7) gives a radiative decay length. Results of τ_{RG} for $I_0 = 10^6$ w/cm² are shown in Fig. 5. The calculation is for the sum of two absorbing bands. Results for other values of incident laser intensity can be directly obtained from Eq. (8). A rather surprising discovery from Fig. 5 is that, for a fixed value of I_0, τ_{RG} is practically constant.

It is of interest to compare τ_{RG} to a characteristic flow time, τ_f. As shown in Ref. 5

$$\tau_f = R^2\rho_\ell/5\,\mu_g \tag{9}$$

where μ_g is the viscosity of the carrier gas. The ratio of the two characteristic times is shown in Fig. 6 for $I_0 = 10^6$ w/cm². As R increases or μ_g decreases, τ_{RG}/τ_f decreases rapidly. Therefore, to have a vaporization zone much smaller than the length of the nozzle needed for accelerating droplets to high speeds, a droplet size $\gtrsim 10\mu$ has to be employed.

The following dimensionless variables are used:

$$\tilde{X} = X/u_0\tau_0 \qquad \tilde{y} = y/d$$

$$f = I/I_0 \qquad g = V/V_0 = (R/R_0)^3$$

For simplicity, the tilde notation will be suppressed from now on and all equations are to be understood as dimensionless.

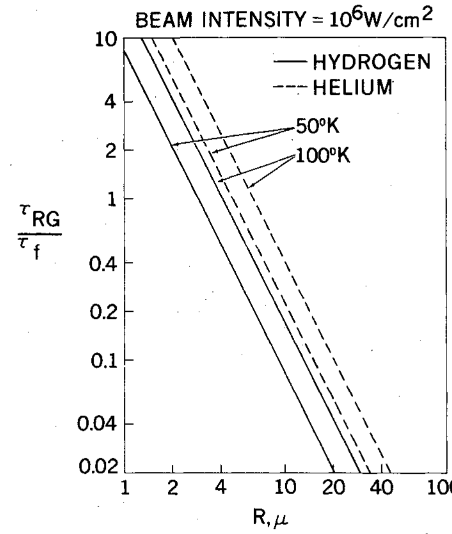

Fig. 6 Ratio of characteristic evaporation time for $\bar{Q}=1$ to characteristic flow time.

The governing equations thus become

$$\partial g/\partial X = -3fg/[1-\epsilon h(y)] \tag{10a}$$

$$\partial f/\partial y = -fg \tag{10b}$$

$$g(0,y) = 1 \tag{11a}$$

$$f(X,0) = 1 \tag{11b}$$

Differentiating Eq. (10b) with respect to X, we obtain

$$\frac{\partial g}{\partial X} = -\frac{\partial}{\partial X}\left(\frac{1}{f}\frac{\partial f}{\partial y}\right) = -\frac{\partial}{\partial y}\left(\frac{1}{f}\frac{\partial f}{\partial X}\right) \tag{12}$$

Substituting Eqs. (10b) and (12) into Eq. (10a), we obtain

$$\frac{\partial}{\partial y}\left(\frac{1}{f}\frac{\partial f}{\partial X}\right) = -\frac{3}{[1-\epsilon h(y)]}\frac{\partial f}{\partial y} \tag{13}$$

The boundary conditions become

$$f(0,y) = e^{-y} \tag{14a}$$

$$f(X,0) = 1 \tag{14b}$$

Perturbation Solutions

Consider the following expansion procedure:

$$f(X,y) = f^{(0)}(X,y) + \epsilon f^{(1)}(X,y) + O(\epsilon^2) \tag{15}$$

Substituting Eq. (15) into Eq. (13) and equating terms of identical powers in ϵ, we obtain

$$\frac{\partial}{\partial y}\left[\frac{1}{f^{(0)}}\frac{\partial f^{(0)}}{\partial X}\right] = -3\frac{\partial f^{(0)}}{\partial y} \tag{16}$$

$$\frac{\partial}{\partial y}\left[\frac{1}{f^{(0)}}\left[\frac{\partial f^{(1)}}{\partial X} - \frac{f^{(1)}}{f^{(0)}}\frac{\partial f^{(0)}}{\partial X}\right]\right] = -3\left[h\frac{\partial f^{(0)}}{\partial y} + \frac{\partial f^{(1)}}{\partial y}\right] \tag{17}$$

Boundary conditions are

$$f^{(0)}(0,y) = e^{-y} \tag{18a}$$

$$f^{(0)}(X,0) = 1 \tag{18b}$$

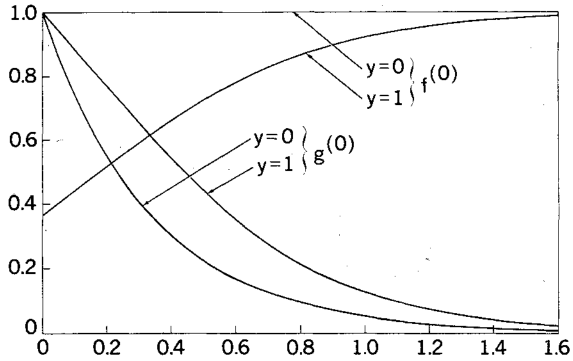

Fig. 7 Uniform-velocity solutions of laser intensity and droplet volume.

$$f^{(1)}(0,y) = f^{(1)}(X,0) = 0 \tag{19}$$

The solution to Eqs. (16) and (18) can be obtained in a straight-forward manner as

$$f^{(0)}(X,y) = 1/[1+(e^y-1)e^{-3X}] \tag{20}$$

Define

$$P(X,y) \equiv \int_0^y h(y') \frac{\partial f^{(0)}(X,y')}{\partial y'} dy' \tag{21}$$

and

$$H(X,y) \equiv \frac{1}{X}\int_0^X \frac{e^{3X'}}{f^{(0)}(X',y)} P(X',y)dX' \tag{22}$$

then the solution to Eqs. (17) and (19) is

$$f^{(1)}(X,y) = -3Xe^{-3X}[f^{(0)}(X,y)]^2 H(X,y) \tag{23}$$

If the dimensionless droplet volume, $g(X,y)$, is expanded in powers of ϵ in a form similar to Eq. (15), the solutions are [from Eqs. (10b)]

$$g^{(0)}(X,y) = e^{y-3X}/[1+(e^y-1)e^{-3X}] \tag{24}$$

and

$$g^{(1)}(X,y) = \frac{f^{(1)}(X,y)}{f^{(0)}(X,y)}\left[g^{(0)}(X,y) - \frac{1}{H(X,y)}\frac{\partial H(X,y)}{\partial y}\right] \tag{25}$$

Therefore, the effect of an arbitrary velocity perturbation on the laser intensity, $f^{(1)}(X,y)$, and the volume of the unvaporized droplet, $g^{(1)}(X,y)$, can be expressed in closed forms. The zeroth-order solutions, Eqs. (20) and (24), are identical to that of Glickler's.[4] This is because, for the case of a uniform velocity, the present two-dimensional steady-state problem can be transformed into an equivalent one-dimensional unsteady problem. It is of interest to note that the forms of Eqs. (10) and (11) are formally equivalent to that of the propagation of a slightly focused laser beam into a stationary cloud of droplets, with $h(y)$ related to the area change of the laser beam. For the laser propagation problem, a reasonable approximation for $h(y)$ away from the focal point is

$$h(y) = y \tag{26}$$

In addition, for this particular choice of $h(y)$, $P(X,y)$ can be explicitly expressed in terms of elementary functions

$$P(X,y) = [1/s(1-e^{-3X})](s \ln s - ye^{y-3X}) \tag{27}$$

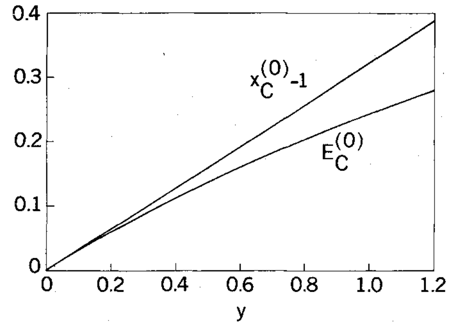

Fig. 8 Critical distance and energy exchange ratio for uniform-velocity case.

where

$$s \equiv 1/f^{(0)}(X,y) = 1+(e^y-1)e^{-3X} \tag{28}$$

and hence $f^{(1)}$ and $g^{(1)}$ involve only single quadratures.

The uniform-velocity solutions for the laser intensity [Eq. (20)] and for the volume of the unvaporized droplet [Eq. (24)] as a function of the distance traveled by the droplet are shown in Fig. 7 at the boundary of the two-phase flow ($y=0$) and at one radiative decay length away ($y=1$). Williams' solution[1] corresponds to $y=0$. Note that, as droplets are vaporized, the attenuation effect due to droplet shielding decreases and, hence, the laser intensity at $y=1$ increases as X increases. One quantity of interest is the distance X_C at which the droplet radius is reduced to e^{-1} of its initial value. Therefore, about 95% of the droplet is vaporized at $X=X_C$, or, more precisely

$$g^{(0)}(X_C,y) + \epsilon\, g^{(1)}(X_C,y) = e^{-3} \tag{29}$$

The critical distance X_C is a function of y. For example, Fig. 7 indicates that, because of droplet shielding, the distance required to vaporize 95% of a droplet at $y=1$ is about 30% longer than that at $t=0$. To investigate the problem further, we may expand X_C in powers of ϵ as

$$X_c = X_C^{(0)} + \epsilon\, X_c^{(1)} \tag{30}$$

Substituting Eq. (30) into Eq. (29), and utilizing the Taylor series expansion, we obtain

$$g^{(0)}(X_C^{(0)},y) = e^{-3} \tag{31}$$

and

$$g^{(1)}(X_C^{(0)},y) + \left[\frac{\partial g^{(0)}}{\partial X}\right]_{X=X_C^{(0)}} X_C^{(1)} = 0 \tag{32}$$

The solution to Eq. (31) is

$$X_C^{(0)} = 1 + \tfrac{1}{3}\ln[e^y + (1-e^y)e^{-3}] \tag{33}$$

As shown in Fig. 8, $X_C^{(0)} - 1$ increases essentially linearly in y.

Another quantity of interest is the energy exchange ratio E_C defined as the ratio of the absorbed to incident energy. As a laser beam propagates into a cloud of droplets, its intensity will be attenuated along its path. Therefore, E_C will depend on the thickness of the particle flow along the beam path y. In addition, it is obvious that the amount of energy absorbed per droplet increases with distance X. To fix ideas, X is chosen to be the critical distance X_C. It is easy to show that

$$E_C = 1 - \frac{1}{X_C}\int_0^{X_C} f(X,y)dX \tag{34}$$

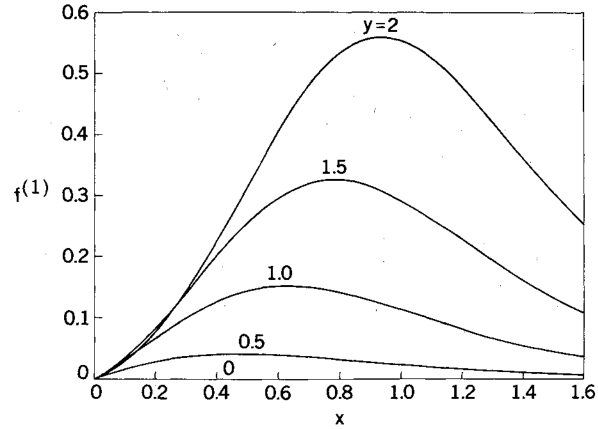

Fig. 9 First-order correction to laser intensity for linearly decreasing velocity.

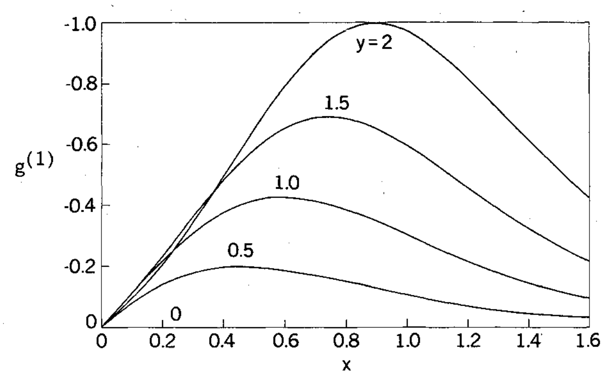

Fig. 10 First-order correction to droplet volume.

If E_C is expanded in powers of ϵ, after some straightforward algebra we may obtain from Eqs. (15), (20), (30), (33), and (34) the following:

$$E_C{}^{(0)} = 1 - 1/X_C{}^{(0)} \tag{35a}$$

$$E_C{}^{(1)} = \frac{1}{X_C{}^{(0)}} \left\{ X_C{}^{(1)} \left[\frac{1}{X_C{}^{(0)}} - f^{(0)}(X_C{}^{(0)}, y) \right] - \int_0^{X_C{}^{(0)}} f^{(1)}(X,y)\,dX \right\} \tag{35b}$$

As shown in Fig. 8, $E_C{}^{(0)}$ increases in y. However, because of droplet shielding, the rate of increase drops as y increases.

Since

$$\partial g^{(0)} / \partial X < 0 \tag{36}$$

according to Eq. (32), $X_C{}^{(1)}$ is of the same sign as $g^{(1)}$. On the other hand, the sign of $E_C{}^{(1)}$ depends on the balance among three terms, namely, the effects of the perturbation in the critical distance on the incident and on the transmitted laser powers, and the perturbation in the transmitted laser power [see Eq. (35b)]. The first-order corrections to the uniform-velocity solutions $f^{(1)}$ and $g^{(1)}$ are explicitly carried out for the particular choice of the velocity-perturbation function, Eq. (26). They are shown in Figs. 9 and 10. Obviously, the corrections are zero at $y=0$. This perturbation of a linearly decreasing droplet velocity in the direction of the laser beam path increases both the power transmitted and the amount of the droplet vaporized. Therefore, $X_C{}^{(1)}$ is also negative. This reduction in the critical distance due to the velocity perturbation is sufficient to overcompensate for the increase in the perturbed transmitted laser intensity to make $E_C{}^{(1)}$ positive. The ratios of the perturbed to the zeroth-order results for E_C and X_C are shown in Fig. 11.

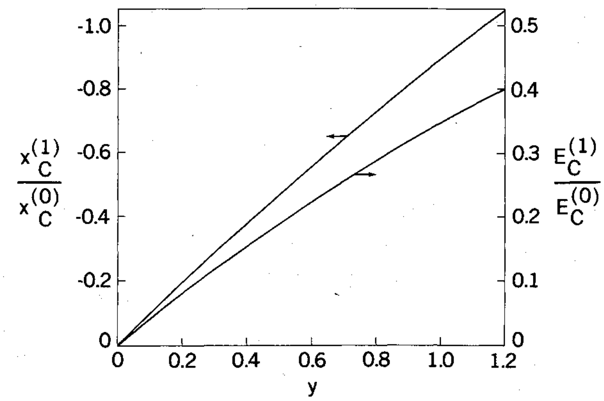

Fig. 11 Ratios of first-order corrections to uniform-velocity solutions for critical distance and energy exchange ratio.

Discussion

It is, perhaps, illustrative to consider the following example. For a laser intensity of 10^6 w/cm^2 and a flow velocity of 10^4 fps, 95% of an air droplet will be vaporized over a distance of about two meters. Although the vertical thickness of the beam can be made much smaller than the beam width, in a steady-state operation, the required laser power is still quite high. An increase in laser intensity will decrease the required beam width but not the total power.

The reason for this large laser power required is obvious. It is because liquid air is a relativly poor absorber. For example, at 60 K, for $R_0 = 10\ \mu$ and a droplet number density of 2.4×10^5/cm^3, the radiative decay length is about seven meters. For a nozzle width of one meter, this yields $y=0.14$. According to Fig. 8, only about 4% of the laser energy is absorbed by the liquid-air droplets. Clearly, more systematic measurements of the radiative properties of liquid air over a wide range of temperatures and pressures are in order before the quantitative results of our example can be considered conclusive.

One may increase the droplet size and/or the droplet number density to shorten the radiative decay length. On the other hand, the nozzle width may also be increased. All of the methods will increase the total nondimensional absorption length y and, hence, the energy exchange ratio. However, the maximum droplet size attainable is limited by a Weber number restriction,[5] whereas an increase in the droplet number density or in the nozzle width will increase the total liquid-air mass-flow rate and hence the total energy required to accelerate these droplets to the desired speed.

A very attractive idea to decrease the characteristic evaporation time (and, hence, the required beam width) is to use strongly absorbing additives. For example, the absorption coefficient of water at 10.6 μ is 1.2×10^3 cm^{-1} (Ref. 3). If only 1% of water is mixed with liquid air, the characteristic evaporation time (and hence the power required) is reduced by a factor of 10 from that of pure liquid air. Note further that by utilizing additives the more readily available high-energy lasers operating at frequencies other than the fundamental vibration-rotation frequencies of liquid O_2 and N_2 can also be advantageously employed. However, this scheme is useful only if the additive is homogeneously mixable with liquid air.

One may apply the analytic solutions developed in the last section to other problems, such as laser propagation in the atmosphere with a variable crosswind or vehicle protection against laser heating by particle injection. However, it might be worthwhile to restate that the elegant closed-form solutions are obtained because of the several simplifying assumptions introduced earlier. For other applications with different materials, these assumptions need to be reexamined. A subtle but important point is that the interpretation of the perturbed solutions may also depend on the particular problem under study. For example, in the present application, we are interested in the critical distance, X_C, where 95% of the droplet is vaporized. A flow velocity linearly *decreasing* in the direction of the laser beam reduces this critical distance and increases the energy exchange ratio. However, for the vehicle-protection problem, for example, the beam width of the incident laser is fixed. Therefore, to decrease the total power transmitted to the vehicle, the present perturbation solution suggests a flow velocity that is linearly *increasing* in the direction of the laser beam.

Conclusions

In summary, we may conclude that the concept of vaporizing liquid-air droplets by laser beams is feasible, although the power required, according to our present knowledge of the radiative properties of liquid air, is high. The idea of mixing strongly absorbing additives to reduce the characteristic evaporation time (and hence total power) appears quite attractive. The effect of droplet shielding is moderate, and a linearly decreasing droplet velocity in the direction of the laser beam can increase the energy exchange ratio somewhat further.

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